Mathematics
Calculation Strategies

A guide to mental and written calculations and other useful strategies we use in our school.

‘To inspire and educate for life’
Introduction

This booklet explains how children are taught to carry out written calculations for each of the four number operations (addition / subtraction / multiplication / division).

In order to help develop your child’s mathematical understanding, each operation is taught according to a clear progression of stages. Generally, children begin by learning how written methods can be used to support mental calculations. They then move on to learn how to carry out and present calculations horizontally. After this, they start to use vertical methods, first in a longer format and eventually in a more compact format (standard written methods). However, we must remember that standard written methods do not make you think about the whole number involved and don’t support the development of mental strategies. They also make each operation look different and unconnected. Therefore, children will only move onto a vertical format when they can identify if their answer is reasonable and if they can make use of related number facts. (Advice provided by Hampshire Mathematics Inspector).

It is extremely important to go through each of these stages in developing calculation strategies. We are aware that children can easily be taught the procedure to work through for a compact written method. However, unless they have worked through all the stages they will only be repeating the procedure and not using it with true mathematical understanding. This can easily lead to mistakes and misconceptions.

The overall aim is that when children leave primary school they:

- have a secure knowledge of number facts and a good understanding of the four operations and to be able to have an efficient, reliable method of calculating each of the four operations
- are able to use this knowledge and understanding to carry out calculations mentally
- make use of diagrams and informal notes to help record steps and part answers when using mental methods that generate more information than they can keep in their heads
- use a calculator effectively

Research has also shown that there are two factors which can make a big difference when children are learning to calculate.

Firstly, it is important to use the correct words when talking about the numbers in calculations. The value of the number should be said. E.g. 94 is 90 (or 9 tens) and 4 units. Secondly, children find it much easier to grasp new methods when given pictures to look at or concrete apparatus to use. Drawings, counters and objects all help.

Please be aware that children will progress at different rates and children will have their own preferred method.
Glossary of Terms

Array: An arrangement of numbers or objects in rows and columns (for teaching of multiplication and division)

Bridging/crossing the 10s boundary: The term used when numbers jump over a multiple of 10 (e.g. $2 + 9 = 11$ this calculation jumps over 10. $39 + 5 = 44$ this calculation jumps over 40)

Chunking: A method for dividing by subtracting by ‘chunks’

Estimate: To roughly calculate the value, number or quantity. Essential to use to determine whether an answer is ‘reasonable’

Equals (Sum of/Total): Meaning two things are the same amount or values connected by the $=$ sign

Factors: Whole numbers that divide exactly into another number

HTU: An acronym for a three digit number (Hundreds, Tens, Units)

Inverse: Doing the opposite or reversing something. Commonly used to check answers by using the inverse to work backwards

Mental methods: Processes carried out predominantly in the head; however, jottings can be used to assist this process

Multiples: The result of multiplying a whole number by another whole number

Number Bonds: A pair of numbers which add to a particular number you are interested in. (e.g. $7 + 3 = 10$)

Number Line: A line (either horizontal or vertical) that is used to aid the children when calculating problems using any of the four operations. The line can be numbers, unnumbered marked or blank – the children typically move through all four stages until they are competent using the blank or empty number line for all calculations

Number sentence: The term given to the calculation

Operations: addition, subtraction, multiplication and division

Place Value: The value of the digit determined by its position in a number

Partitioning: This is a term used to describe the process of ‘splitting’ or breaking up numbers into hundreds, tens and units (e.g. 126 = 1 hundred, 2 tens and 6 units or, 100 + 20 + 6)

Recombining: This is the term used to describe the process of putting partitioned numbers back together
We use the term ‘number sentence’ rather than sum. This is because the term ‘sum’ means the result of adding two or more numbers together.
Mental Skills

Written methods of calculations are based on mental strategies. Each of the four operations builds on mental skills which provide the foundation for jottings and informal written methods of recording. Skills need to be taught, practised and reviewed constantly. These skills lead on to more formal written methods of calculation.

**Addition**
Recognise the size and position of numbers
Count on in ones, tens, hundreds, thousands, decimals
Know number bonds to 10, 20, 100 and beyond
Add multiples of 10 to any number
Partition and recombine numbers
(e.g. 57 = 50 + 7)
Bridge through the tens barrier

**Subtraction**
Recognise the size and position of numbers
Count back in ones, tens, hundreds, thousands, decimals
Know number facts for all numbers to 10, 20, 100 and beyond
Subtract multiples of 10 from any number
Partition and recombine numbers (only split the number to be subtracted)
Bridge through the tens barrier

**Multiplication**
Recognise the size and position of numbers
Count on in different steps 10s, 5s, 2s, 4s, 8s, 3s, 6s, 9s and 7s
Double numbers up to 10 and beyond
Recognise multiplication as repeated addition
Quick recall of multiplication facts (times tables)
Use known facts to derive associated facts (e.g. 2 x 4 = 8, so 20 x 4 = 80)
Multiplying by 10, 100, 1000 and understanding the effect
Multiplying by multiples of 10

**Division**
Recognise the size and position of numbers
Count back in different steps 2s, 5s, 10s, 100s, 1000s, decimals
Halve numbers to 20 and beyond
Recognise division as repeated subtraction
Quick recall of division facts
Use known facts to derive associated facts
Divide by 10, 100, 1000 and understanding the effect
Divide by multiples of 10
When we introduce new concepts, we use familiar objects and resources, to reinforce the children’s understanding. When teaching a new strategy it is important to start with numbers that the child can easily use so that they can understand the concept.

Counting apparatus – cubes, counters and bead strings

Counting stick

Place value apparatus – e.g. Dienes apparatus

Place value cards

Number lines

Hundred squares and Multiplication grids

Arrays

4 x 3 = 12
Progression in teaching addition:

The number line method

Children need to know that addition can be done in any order.

Add two single-digit numbers that bridge 10.

Begin to partition numbers in order to add.

Adding two two-digit numbers (without bridging)
Counting in tens and ones partitioning and recombinining.

15 + 13 = 28
Addition

Adding two two-digit numbers (bridging through tens boundary)

Using a number line

Or

Using place value cards and place value apparatus to partition numbers and recombine

48 + 36 = 84

Movement to standard methods:

Expanded method
The expanded method enables children to see what happens to numbers in the standard written method

48 + 36

As we can’t have 14 units, the children would then add 70 + 10 + 4, to give the answer of 84

T U
4 8
+ 3 6
1 4
7 0
8 4

Standard written method
The previous stages reinforce what happens to the numbers when they are added together using more formal written methods

48 + 36
84
1
Progression in teaching subtraction:

The number line method

Subtract single digit numbers of ten bridging through 10.

Children to use known facts and number bond knowledge to make manageable jumps

\[ 15 - 7 = 8 \]

\[ 8 + 2 \quad +5 \quad 10 \quad 15 \]

The difference between 11 and 14 is 3.

\[ 14 - 11 = 3 \]
\[ 11 + 3 = 14 \]

Begin to find the difference by counting up from the smallest number

Begin to partition numbers in order to take away

Decide whether to count on or count back

\[ 74 - 27 = 47 \]
Subtraction

43 - 7 = 8

15 - 7 = 8

Movement to standard methods:

43 - 27 = 16

to subtract 7 units we need to exchange a ten for ten units

138
-74
---
6 = 80 (74 + 6 = 80)
20 = 100 (80 + 20 = 100)
30 = 130 (100 + 30 = 130)
8 = 138 (130 + 8 = 138)

Or

30
+ 10 + 3
---
20 + 7
---
10 + 6
---

Standard written method

The previous stages reinforce what happens to the numbers when they are subtracted using more formal methods. It is important that the children have a good understanding of place value and partitioning.
Progression in teaching multiplication:

- **2 x 4**
  - $2 \times 4 = 8$
  - $4 \times 2 = 8$

- **4 x 2**
  - $4 \times 2 = 8$
  - $2 \times 4 = 8$

Understand multiplication as an array.

Understand how to represent arrays on a number line.

Use place value apparatus to support the multiplication of U x TU.

Use place value apparatus to support the multiplication of U x TU alongside the grid method.

$40 + 12 = 52$
Multiplication

The grid method

Multiplying TU x TU using the 'grid method'

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<th>30</th>
</tr>
</thead>
<tbody>
<tr>
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<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>120</td>
<td>12</td>
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</tbody>
</table>

= 330 +
= 132

462

Note:
Children are to use their taught method for addition to complete this multiplication procedure e.g. the number line or vertical method.
Division

Progression in teaching division:

Understand division as grouping

12 divided into groups of 3 gives 4 groups

12 ÷ 3 = 4

Reinforce division as grouping through the use of arrays

12 divided into groups of 4 gives 3 groups

12 ÷ 4 = 3

Represent ‘groups’ for division on a number line using apparatus alongside the line

18 divided into groups of 3

18 ÷ 3 = 6

The number line method
Division

Movement to standard methods:

The chunking method

Children need to see that as the numbers get larger, large chunk subtraction is the more efficient method. Multiples of the divisor (large chunks) are taken away. Multiplication facts are needed to see the size of the ‘chunk’.

518 ÷ 7 = 74

100 ÷ 7 = 14 r 2

560 ÷ 24 = 23 r 8

Hello

Hints:

1 x 7 = 7
2 x 7 = 14
5 x 7 = 35
10 x 7 = 70
20 x 7 = 140
50 x 7 = 350
100 x 7 = 700

The children are encouraged to write a ‘Handy Hints’ box

Standard written method

This links directly to large/efficient chunk subtraction
Dealing with Decimals

The methods used to solve calculations may or may not vary when children are required to calculate using decimals.

Adding decimals:

Both number lines and more standard (vertical) methods for addition (as above) can be used when adding decimals.

Example: 3.2 + 16.3 =

The number line method

The expanded method

| 3.2 |
| + 16.3 |
| 0.5 |
| 9.0 |
| 10.0 |
| 19.5 |

The standard method

3.2
+ 16.3
19.5

Subtracting decimals:

Both number lines and more standard (vertical) methods for addition (as above) can be used when adding decimals.

Example: 19.5 – 16.3 =

The number line method (by counting on)

The standard method

19.5
- 16.3
3.2
Dealing with Decimals

Multiplying decimals:

When multiplying with decimals, the grid method is used.

Example: \(33.2 \times 14.4 = \)

<table>
<thead>
<tr>
<th></th>
<th>30</th>
<th>3</th>
<th>0.2</th>
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<td>12</td>
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</tr>
<tr>
<td>0.4</td>
<td>12</td>
<td>1.2</td>
<td>0.08</td>
</tr>
</tbody>
</table>

\[= 332\] \[= 132.8 +\] \[= 13.28\] \[\text{478.08}\]

_Note:_ Children are to use their taught method for addition to complete this multiplication procedure e.g. the number line or vertical method.

Dividing decimals:

When presenting remainders in decimal form, short division is used.

Example: \(560 \div 24 = \)

\[23.333 \quad \overline{\sqrt{560.000}}\]

_Note:_ Presenting remainders in decimal form is a complex skill which will only be taught to those children approaching ‘level 6’ mathematics.
Helping at Home

Just remember the 3Cs:

Cooking

What can be weighed, measured, estimated and compared at home?

(Metric units of measure used in the lower school. Children in the upper school need to use metric units and be aware of imperial units of measure)

Clocks

How many clocks are there in your house? Are they digital?
Are they analogue?
Can your child read both?

Coins

Hand that shopping list over to your children – can they work out your change?

We hope that this booklet has provided you with a helpful insight into the way calculation strategies are taught in school.

If you have any questions, please contact your child’s maths teacher.
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