Mathematics
Calculation Strategies

A guide to mental and written calculations strategies we use in our school.

'To inspire and educate for life'

Introduction

This booklet explains how children are taught to carry out written calculations for each of the four number operations (addition / subtraction / multiplication / division) at Droxford Junior School.

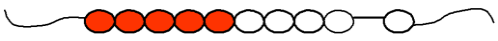
In order to help develop your child's mathematical understanding, each operation is taught according to a clear progression of stages. Generally, children begin by learning how written methods can be used to support mental calculations. They then move on to learn how to carry out and present calculations horizontally. After this, they start to use vertical methods, first in a longer format and eventually, in a more compact format (standard written methods). However, we must remember that standard written methods do not make you think about the number involved and don't support the development of mental strategies. They also make each operation look different and unconnected. Therefore, children will only move onto a vertical format when they can identify if their answer is reasonable and if they can make use of related number facts.

It is extremely important to go through each of these stages in developing calculation strategies. We are aware that children can easily be taught the procedure to work through for a compact written method. However, unless they have worked through all the stages, they will only be repeating the procedure and not using it with true mathematical understanding. This can easily lead to mistakes and misconceptions.

Our overall aim is that when children leave Droxford Junior School they:

- have a secure knowledge of number facts and a good understanding of the four operations and have an efficient, reliable method of calculating each of the four operations;
- are able to use this knowledge and understanding to carry out calculations mentally; and
- make use of diagrams and informal notes to help record steps and part answers when using mental methods that generate more information than they can keep in their heads.

Research has also shown that there are two factors which can make a big difference when children are learning to calculate.
Firstly, it is important to use the correct words when talking about the numbers in calculations. The value of the number should be said. E.g. 94 is 90 (or 9 tens) and 4 units.
Secondly, children find it much easier to grasp new methods when given pictures to look at or concrete apparatus to use. Drawings, counters and objects all help.



Please be aware that children will progress at different rates and children will have their own preferred method at different stages in their learning.

Glossary of Terms

Array: An arrangement of numbers or objects in rows and columns (for teaching multiplication and division)

Bridging/crossing the 10s boundary: The term used when numbers jump over a multiple of 10 (E.g. 2 + 9 = 11 this calculation jumps over 10. 39 + 5 = 44 this calculation jumps over 40)

Dividend: The amount that you want to divide up. (E.g. dividend ÷ divisor = quotient so in 12 ÷ 3 = 4, 12 is the dividend)

Divisor: The number you divide by

Estimate: To roughly calculate the value, number or quantity. Essential to use to determine whether an answer is 'reasonable'

Equals (Sum of/Total): Meaning two things are the same amount or values connected by the = sign

Factors: Whole numbers that divide exactly into another number

HTU: An acronym for a three digit number (Hundreds, Tens, Units)

Inverse: Doing the opposite or reversing something. Commonly used to check answers by using the inverse to work backwards

Mental methods: Processes carried out predominantly in the head; however, jottings can be used to assist this process

Multiples: The result of multiplying a whole number by another whole number

Number Bonds: A pair of numbers which add to a particular number you are interested in (E.g. 7 + 3 = 10)

Number Line: A line (either horizontal or vertical) that is used to aid the children when calculating problems using any of the four operations. The line can be numbered, unnumbered, marked or blank – the children typically move through all four stages until they are competent using the blank or empty number line for all calculations

Number sentence: The term given to the calculation (E.g. 26 + 4 = 30)

Operations: Addition, subtraction, multiplication and division

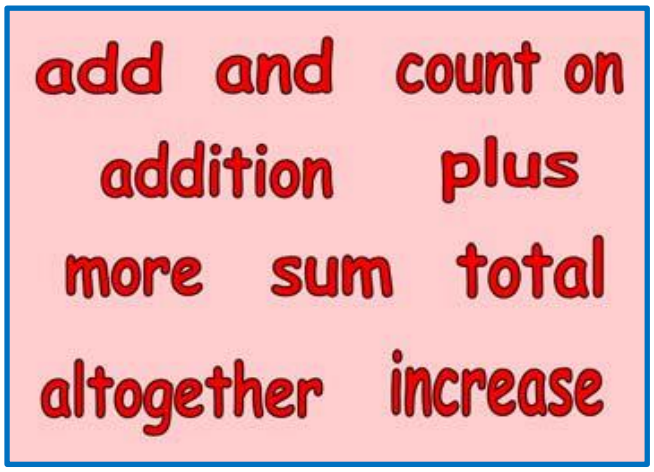
Place Value: The value of the digit determined by its position in a number

Partitioning: This is a term used to describe the process of 'splitting' or breaking up numbers into hundreds, tens and units (E.g. 126 = 1 hundred, 2 tens and 6 units or, 100 + 20 + 6)

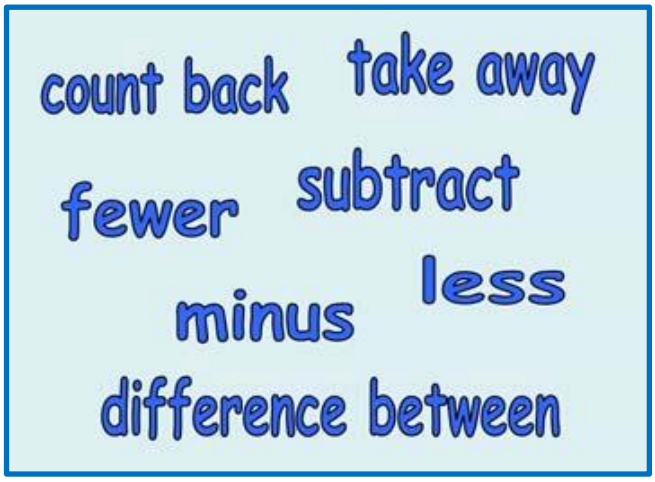
Recombining: This is the term used to describe the process of putting partitioned numbers back together

Key Vocabulary

Words for addition:

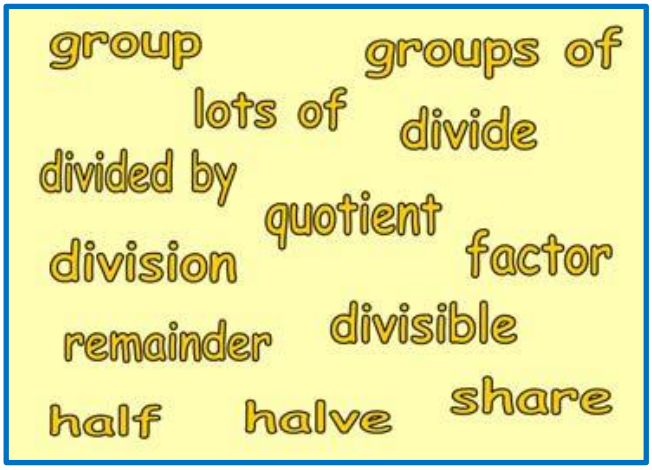


Words for subtraction:



Words for multiplication:

Words for division:



We use the term ‘number sentence’ rather than sum. This is because the term ‘sum’ means the result of adding two or more numbers together.

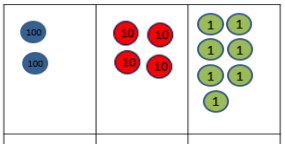
Models and Images

When we introduce new concepts, we use familiar objects and resources to reinforce the children's understanding.
At home, you could use similar models to support your child's understanding.

Counting apparatus – cubes, counters and bead strings

Counting stick

Place value apparatus – Dienes apparatus or place value counters

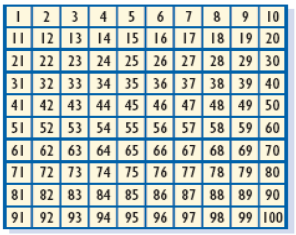


Place value cards

or

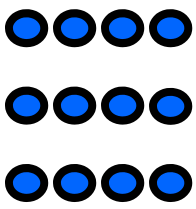
Number lines

Hundred squares and Multiplication grids



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Arrays



4 x 3 = 12

Mental Skills

Written methods of calculations are based on mental strategies. Each of the four operations builds on mental skills which provide the foundation for jottings and informal written methods of recording. Skills need to be taught, practised and reviewed constantly. These skills lead on to more formal written methods of calculation.
Below are high order mental skills children need to be working towards.

Addition

Know by heart number bonds to 100 and use these to derive related facts. (E.g. 3.46 + 0.54 = 4)
Derive quickly, and without difficulty, number bonds to 1000
Add small and large whole numbers where the use of place value or number facts makes the calculation achievable 'in our heads'. (E.g. 34,000 + 8000)
Add multiples of powers of ten and near multiples of the same. (E.g. 6345 + 199)
Add negative numbers in a context such as temperature where the numbers make sense
Add two 1-place decimal numbers or two 2-place decimal numbers less than 1. (E.g. 4.5 + 6.3 or 0.74 + 0.33)
Add positive numbers to negative numbers. (E.g. calculate a rise in temperature, or continue a sequence beginning with a negative number)
Use place value and number facts to add two or more numbers, including money and decimals. (E.g. 3 + 8 + 6 + 4 + 7, 0.6 + 0.7 + 0.4, or 2,056 + 44)

Subtraction

Use number bonds to 100 to perform mental subtraction of any pair of integers by complementary addition. (E.g. 1000 – 654 as 46 + 300 in our heads)
Use number bonds to 1 to 10 to perform mental subtraction of any pair of 1-place or 2-place decimal numbers using complementary addition and including money. (E.g. 10 – 3.65 as 0.35 + 6, £50 – £34.29 as 71p + £15)
Use number facts and place value to perform mental subtraction of large numbers or decimal numbers with up to two places. (E.g. 467,900 – 3,005 or 4.63 – 1.02)
Subtract multiples of powers of ten and near multiples of the same
Subtract negative numbers in a context such as temperature where the numbers make sense

Mental Skills

Multiplication

Know by heart all the multiplication facts up to 12 x 12
Multiply whole numbers and decimals with up to three places by 10, 100 or 1000. (E.g. 234 x 1000 = 234,000 and 0.23 x 1000 = 230)
Identify common factors, common multiples and prime numbers and use factors in mental multiplication. (E.g. 326 x 6 is 652 x 3 which is 1956)
Use place value and number facts in mental multiplication. (E.g. 40,000 x 6 = 240,000 and 0.03 x 6 = 0.18)
Use doubling and halving as mental multiplication strategies, including to multiply by 2, 4, 8, 5, 20, 50 and 25. (E.g. 28 x 25 is ¼ of 28 x 100 = 700)
Use rounding in mental multiplication. (E.g. 34 x 19 as (20 x 34) – 34)
Multiply 1- and 2-place decimals by numbers up to and including 10 using place value and partitioning. (E.g. 3.6 x 4 is 12 + 2.4 or 2.53 x 3 is 6 + 1.5 + 0.09)
Double decimal numbers with up to 2 places using partitioning. (E.g. 36.73 doubled is double 36 (72) plus double 0.73 (1.46))

Division

Know by heart all the division facts up to 144 ÷ 12
Divide whole numbers by powers of 10 to give whole number answers or answers with up to three decimal places
Identify common factors, common multiples and prime numbers and use factors in mental division. (E.g. 438 ÷ 6 is 219 ÷ 3 which is 73)
Use tests for divisibility to aid mental calculation
Use doubling and halving as mental division strategies, including to divide by 2, 4, 8, 5, 20 and 25. (E.g. 628 ÷ 8 is halved three times: 314, 157, 78.5)
Divide 1- and 2-place decimals by numbers up to and including 10 using place value. (E.g. 2.4 ÷ 6 = 0.4 or 0.65 ÷ 5 = 0.13, £6.33 ÷ 3 = £2.11)
Halve decimal numbers with up to 2 places using partitioning
(E.g. Half of 36.86 is half of 36 (18) plus half of 0.86 (0.43))
Know and use equivalence between simple fractions, decimals and percentages, including in different contexts
Recognise a given ratio and reduce a given ratio to its lowest terms

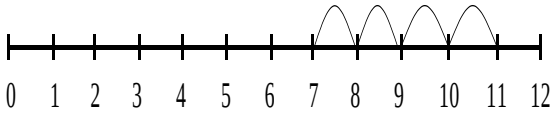
Addition

Progression in teaching addition

The number line method:

Children need to have an understanding of counting on with a number line (supported by models and images).Children also need to know that addition can be done in any order.

E.g. 7 + 4 =



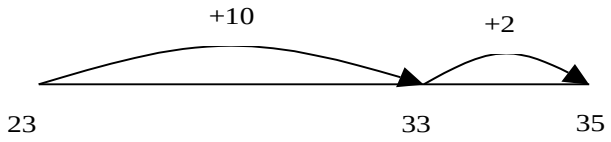
We continue to use number lines to develop understanding of:

<u>Counting on in tens and ones</u>

E.g. 23 + 12 = 23 + 10 + 2

= 33 + 2

= 35

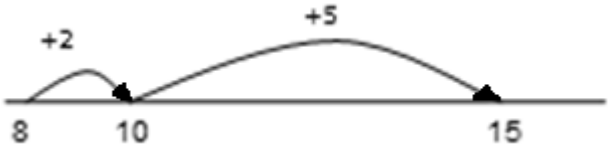


<u>Partitioning and bridging through 10</u>

The steps in addition often bridge through a multiple of 10.

E.g. Children should be able to partition the 7 to relate adding the 2 and then the 5.

8 + 7 = 15



<u>Adding two 2-digit numbers (bridging through tens boundary)</u>

Using a number line or using place value cards and place value apparatus to partition numbers and recombine.

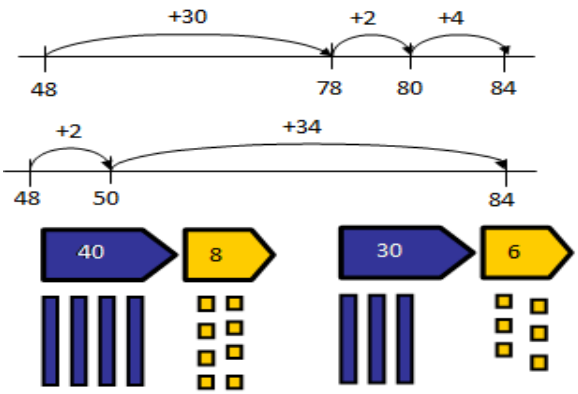
E.g. 48 + 36 = 84

<u>40 + 30 + 8 + 6</u>

40 + 30 = 70

8 + 6 = 14

70 + 14 = 84



Addition

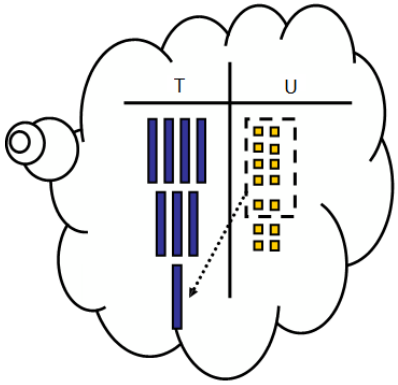
Movement to standard methods:

We introduce expanded column addition which enables children to see what happens to numbers in the standard written method. This can be modelled with place value counters or Dienes so that children can see how it works.

E.g. 48 + 36

As we can't have 14 units, the children would then add 70 + 10 + 4, to give the answer of 84

T	U	
4	8	+
3	6	
1	4	
7	0	+
8	4	



Formal written method of addition:

The previous stages reinforce what happens to the numbers when they are added together using more formal written methods. Some children (National Curriculum Year 3 expectations) may begin to use a formal columnar algorithm, initially introduced alongside the expanded method. The formal method should be seen as a more streamlined version of the expanded method, not a new method.

E.g.

```
  48
+ 36
  84
   1
```

Children will move on to the formal columnar method for whole numbers and decimal numbers as an efficient written algorithm.

E.g.

```
  72.8
+ 54.6
 127.4
 1 1
```

```
  172.83
+  54.68
  227.51
  1 1 1
```

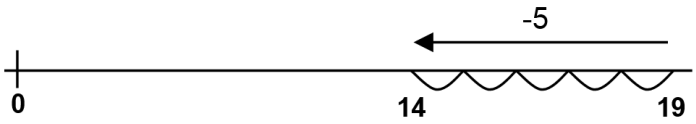
Place value counters can be used alongside the columnar method to develop understanding of addition with decimal numbers.
Finally, children progress to larger numbers, aiming for both conceptual understanding and procedural fluency with columnar method to be secured. They will continue calculating with decimals, including those with different numbers of decimal places.

Subtraction

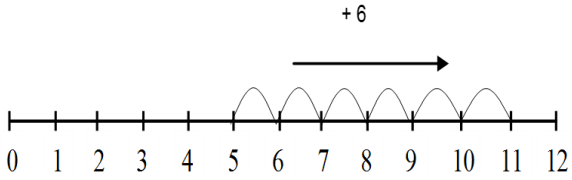
Progression in teaching subtraction

The number line method:

Children need to understand subtraction as take-away:

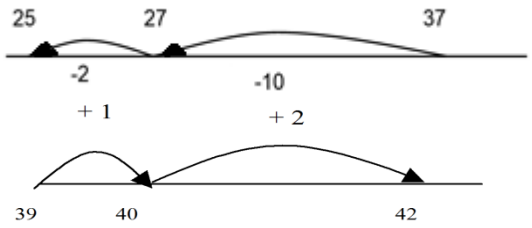


Children also need to understand subtraction as finding the difference:

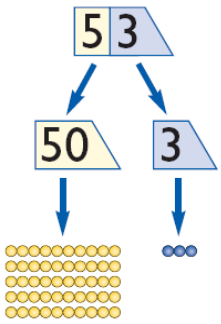


It is valuable to use a range of representations. Children continue to use number lines to model take-away and difference.

E.g.



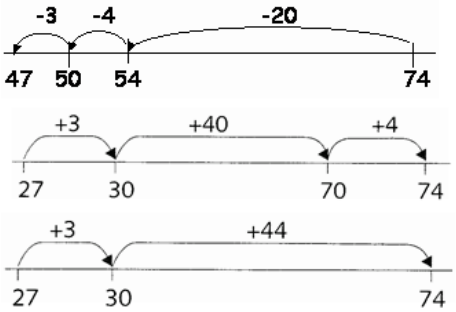
At this stage, children begin to partition numbers in order to take away. E.g



The link between the two may be supported by an image like this, with 47 being taken away from 72, leaving the difference, which is 25.

E.g. 74 – 27 = 47

Children can decide whether to count on or count back

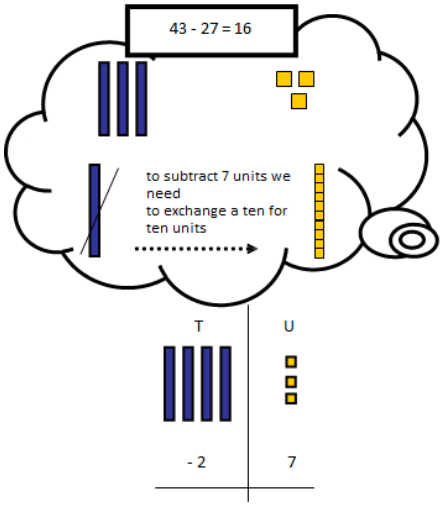


Subtraction

Movement to standard methods:

We introduce expanded column subtraction (this is where the number is expanded out into 100, 10s and units etc. see below for example), initially with no decomposition. Once children are confident with the procedure, we introduce crossing tens boundaries in this format, modelled visually with place value counters or Dienes. This method enables children to see what happens to numbers in the standard written method.

E.g.



$$\begin{array}{rcr} 30\cancel{4} & + & 10+\cancel{3} \\ 20 & + & 7 \\ \hline 10 & + & 6 \end{array} \; -$$

If understanding of the expanded method is secure, children will move on to the formal method of decomposition, which again can be initially modelled with place value counters.

Formal written method of subtraction:

The previous stages reinforce what happens to the numbers when they are subtracted using more formal methods. It is important that the children have a good understanding of place value and partitioning. If children have this, they may begin to use a formal columnar algorithm, initially introduced alongside the expanded method. (National Curriculum Year 3 expectations.) The formal method should be seen as a more streamlined version of the expanded method, not a new method.

E.g.

$$\begin{array}{r} {}^{8}\cancel{9}\,{}^{12}\cancel{3}\,{}^{1}2 \\ 4\;5\;7 \; - \\ \hline 4\;7\;5 \end{array}$$

Finally, children progress to larger numbers, aiming for both conceptual understanding and procedural fluency with columnar method to be secured. They will continue calculating with decimals, including those with different numbers of decimal places.

Multiplication

Progression in teaching multiplication

Children begin by understanding multiplication as an array to see that it can be done in any order (commutative).

2 x 4

$2 \times 4 = 8$

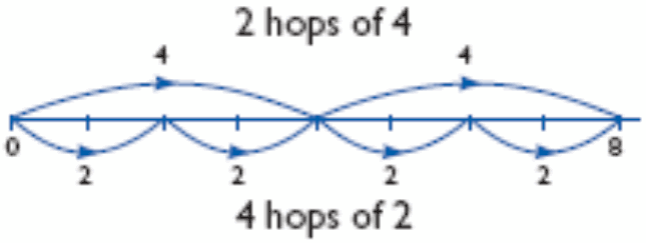
$4 \times 2 = 8$

4 x 2

$4 \times 2 = 8$

$2 \times 4 = 8$

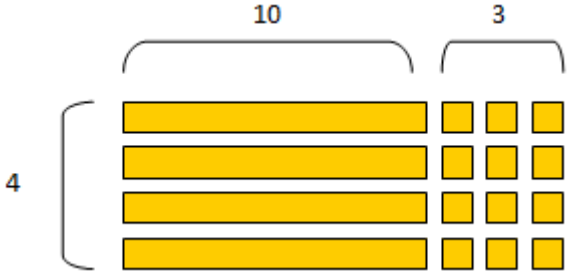
Children then develop their understanding of how to represent arrays on a number line.



Initially children use place value apparatus to support the multiplication of U x TU which allows children to see that they can actually add the numbers together to get the right answer.

E.g.

4 x 13 or 13 x 4



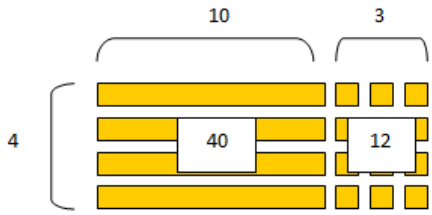
40
12 +
52

Multiplication

Movement towards standard methods

The grid method:

We use place value apparatus (modelled with place value counters or Dienes) to support the multiplication of U x TU alongside the grid method when introduced.



	10	3
4	40	12

40
12 +
52

Children embed and deepen their understanding of the grid method to multiply. We ensure this is still linked back to their understanding of arrays before transition to formal written methods.

Formal written methods of multiplication

Short and Long multiplication:

The previous stages reinforce what happens to the numbers when they are multiplied using more formal methods. Children explore how the grid method supports an understanding of long and short multiplication. (National Curriculum Year 5 & 6 expectations.)

E.g. (for long multiplication)

18 x 13 =

	10	8
10	100	80
3	30	24

18
X 13
54
180 +
234

Note:
Children use their taught method for addition to complete this multiplication procedure.

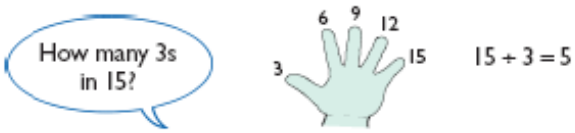
Children continue to refine and deepen their understanding of written methods including fluency for using long multiplication.

Division

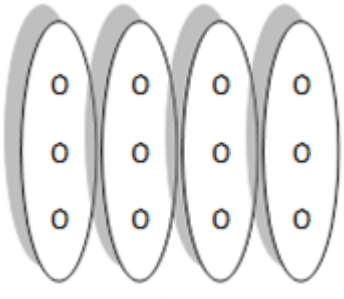
Progression in teaching division

Grouping:

Children should apply their counting skills to develop some understanding of grouping.



Children will use arrays as a pictorial representation for division.



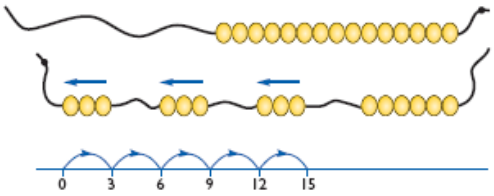
12 divided into groups of 3 gives 4 groups

12 ÷ 3 = 4

Note:
Children should be able to find ½ and ¼ and simple fractions of objects, numbers and quantities.

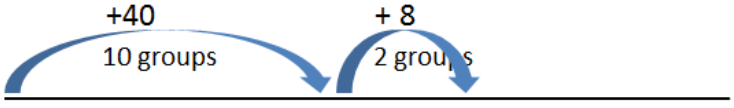
The number line method:

Children will progress to grouping on a number line. E.g.
Group from zero in jumps of the divisor to find out 'How many groups of 3 (the divisor) are there in 15?'
E.g. 15 ÷ 3 = 5



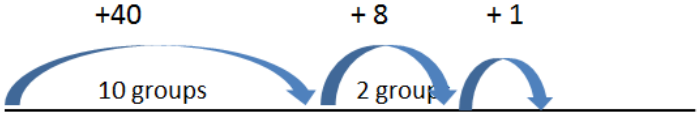
In order to become more efficient using a number line, children need to be able to partition a number in different ways to find out how many groups of 4 are there in 48 (the dividend).

E.g. **48 ÷ 4 = 12**



Remainders

49 ÷ 4 = 12 r1

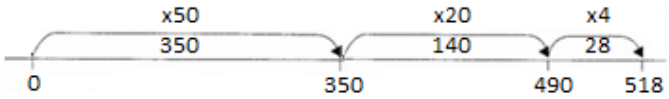


Division

Movement to standard methods

Children will continue to explore division as sharing and grouping, and to represent calculations on a number line until they have a secure understanding**.**
Formal short division will only be introduced once children have a good understanding of division, its links with multiplication and the idea of 'chunking up' to find a target number. (Government guidelines recommend children in year 5 progress to this method.)
Children need to see that as the numbers get larger, large chunk subtraction is the more efficient method. Multiples of the divisor (large chunks) are taken away. Multiplication facts are needed to see the size of the 'chunk'.

E.g. 518 ÷ 7 = 74



The previous stages reinforce what happens to the numbers when they are multiplied using more formal methods. (National Curriculum Year 5 & 6 expectations.)

Short Division:

E.g. 432 ÷ 5 = 86 r 2

$$\begin{array}{r} 8\ 6\ r2 \\ 5\ \overline{)\ 4\ 3^{3}2} \end{array}$$

In this way, children begin to develop their understanding of how to express the remainder as a decimal or a fraction. (National Curriculum Year 5 & 6 expectations.)

Long division:

E.g.

Or 432 ÷ 15 = **28 r 12**
432 ÷ 15 = **28 4/5**

15 | 432

300- **(15 x 20)**

132

120- **(15 x 8)**

12

Note:
We encourage children to make multiplication jottings to support their division.

Or 432 ÷ 15 = **28.8**

2 8 . 8
15 | 4 3 2 . 0
3 0
1 3 2
1 2 0
1 2 0
1 2 0

Dealing with Decimals

The methods used to solve calculations may or may not vary when children are required to calculate using decimals.

Adding decimals:

E.g. 172. 83 + 54.68 =

Formal written method of addition:

$$\begin{array}{r} 172.83 \\ +\ 54.68 \\ \hline 227.51 \\ 1\quad\ 1\ \ 1 \end{array}$$

Subtraction decimals:

E.g. 172. 83 – 54. 68 =

Formal written method of subtraction:

$$\begin{array}{r} 1\ {}^{6}\!\not{7}\ {}^{1}2.\ {}^{7}\!\not{8}\ {}^{1}3 \\ 5\ \ 4.\ \ 6\ 8 \\ \hline 1\ 1\ \ 8.\ 1\ \ 5 \end{array}$$

Dealing with Decimals

Multiplying decimals:

When multiplying with decimals, short multiplication is used.

E.g. 2.4 x 6 =

$$\begin{array}{r} 2.\ 4 \\ \times\ \ 6 \\ \hline 14.\ 4 \\ {\scriptstyle 2} \end{array}$$

Note: Children are to use their taught method for addition to complete this multiplication procedure. (E.g. the number line or vertical method.)

Dividing decimals:

When presenting remainders in decimal form, short division or long division is used.

E.g. 560 ÷ 24 =

$$\begin{array}{r} 23.333 \\ 24\overline{)\,560.000} \\ {\scriptstyle 5\ 8\ 8\ 8\ 8} \end{array}$$

E.g. 432 ÷ 15 = 28.8

$$\begin{array}{r} 28.8 \\ 15\overline{)\,432.0} \\ 30 \\ \hline 132 \\ 120 \\ \hline 120 \\ 120 \\ \hline 0 \end{array}$$

Helping at Home

Just remember the 3Cs:

What can be weighed, measured, estimated and compared at home?

(Metric units of measure used in the lower school. Children in the upper school need to use metric units and be aware of imperial units of measure.)

Cooking

Clocks

How many clocks are there in your house? Are they digital? Are they analogue?

Can your child read both?

Coins

Hand that shopping list over to your child/ren – Can they work out your change?

We hope that this booklet has provided you with a helpful insight into the way calculation strategies are taught in school.

If you have any questions, please contact your child's maths teacher.

‘To inspire and educate for life’